Caustics in Non-linear Compton Scattering

Aleksei Kalinov[1, †], V. Yu. Kharin and S. G. Rykovanov[1, ‡]

[1]HiperCore, Skolkovo Institute of Science and Technology, Moscow, Russia

[†]Aleksei.Kalinov@skoltech.ru, [‡]S.Rykovanov@skoltech.ru

Skoltech
Skolkovo Institute of Science and Technology

An Important Problem

Non-linear Compton Scattering is a prospective source of bright, narrow-band and collimated x rays and gamma rays. A precise theoretical description of its spectrum is tricky and one usually has to rely on numerical computations. Yet this numerical analysis can be computationally expensive due to several parameters affecting the resulting spectra. We show how to harness a stationary phase method to predict caustics in Compton scattered light and extend the results of [1] to linearly-chirped laser pulses with energy conserving varying amplitudes.

Model

We consider a short, linearly-chirped pulse characterized by the following quantities, defined in a frame of reference where electron is initially at rest. Let's denote *time* as τ, *scale factor* as σ, *chirp value* as β and *adjusted chirp value* as $\chi = \frac{\beta}{\sigma^2}$.

Envelope: $E(\tau, \beta) = a_0 \frac{1}{\sqrt[4]{1+\chi^2}} \exp\left\{-\left(\frac{\tau}{2\sigma}\right)^2 \frac{1}{1+\chi}\right\}$

Phase: $\Phi(\tau, \beta) = -\tau^2 \frac{\chi}{2\sigma^2(1+\chi)} + \tau + 0.5arctg\,(\chi)$

We are interested in optimizing the narrowness and brightness of a spectrum integral

$$\frac{d^2 I}{d\omega d\Omega} \sim \left| \int_{-\infty}^{+\infty} \vec{n} \times [\vec{n} \times \vec{u}] \exp\left(i\omega(\tau + z - \vec{n}\vec{r})\right) d\tau \right|^2 \qquad (1)$$

Ray Surfaces

Using Jacobi-Anger expansion and several trigonometric approximations that are valid for short pulse, we derive the equations for ray surfaces of a linearly chirped pulse in cylindrical coordinates with ω as a radius, τ as height and θ as angle:

$$\omega = m \frac{1 - \frac{\chi\tau}{\sigma^2(1+\chi)}}{1 + u_z(1 - \cos\theta)}, \qquad (2)$$

where m is an integer number of a particular harmonics.

Projection mapping along the height allows us to find the singularities, corresponding to cases where higher-order corrections to the spectrum integral 1 are required. Singularities in projection map are caustics in the spectrum!

Visual results

The figure below shows ray surfaces (a), (b), projection maps with singularities (c), (d), and numerically computed spectra (e), (f) for pulses with different chirps. The left column corresponds to $\beta = 0.0$ whereas the right column corresponds to $\beta = 600$.

The projections maps (c), (d) clearly show the difference between non-chirped and chirped pulses, where cusps are present for the latter in addition to folds in the inner oval. These cusps correspond to the regions of the brightest spectrum as seen in the pictures (e), (f) where red cross indicates the maximum value of the spectrum across all angles and frequencies.

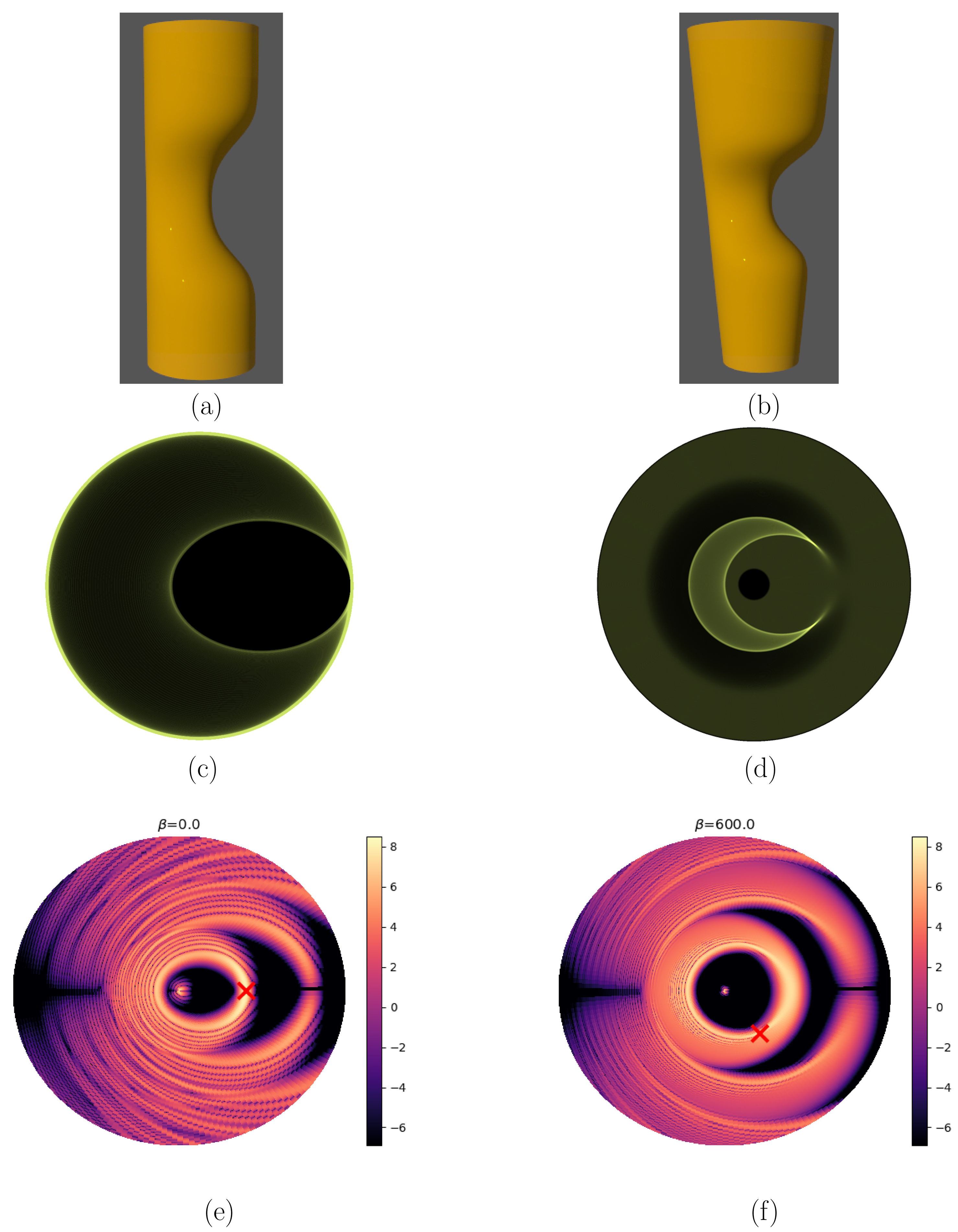
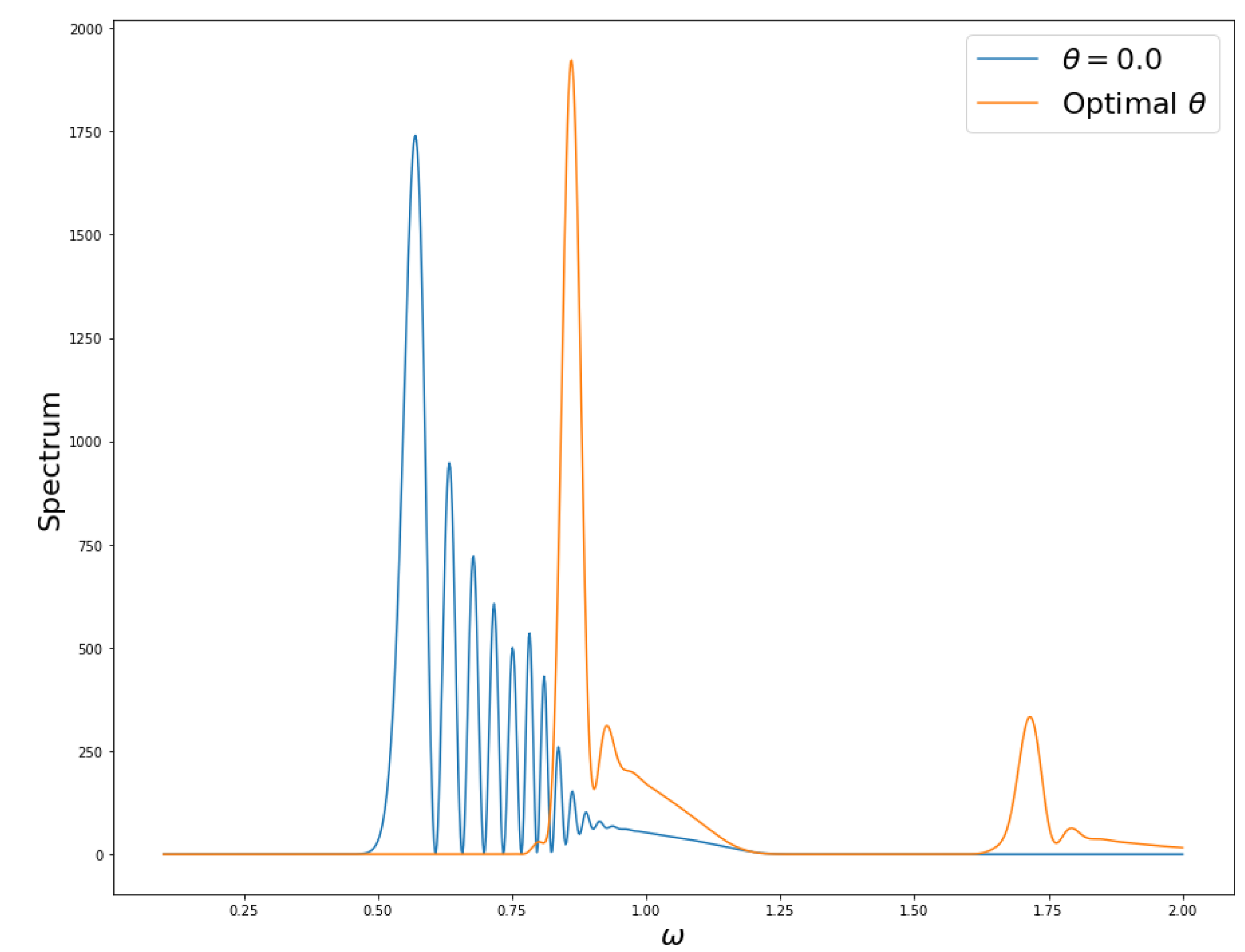


Fig. 1: Ray surfaces, approximated caustics and numerically computed spectra for pulses with different chirps.

Spectrum comparison

We show that singularities in projection mapping that correspond to higher dimensional caustics in spectrum integral are valuable candidates for optimal parameter consideration.

In the figure below that depicts spectra for linearly-chirped pulse with $\beta = 600$ it is visible that a value of θ parameter that is set to align with projection map singularity (optimal) yields much more narrow and brighter rays compared to the regular direction of $\theta = 0.0$, which coincides with the maximum when $\beta = 0.0$.



(a) 0.0

Fig. 2: Comparison of spectrums for linearly-chirped pulse with $\beta = 600$.

Conclusion

We show how geometrical methods of finding singularities in projection of ray surfaces can be used to optimize the narrowness and brightness of a non-linear Compton spectrum, allowing to reduce parameter search space and spare valuable computational resources. The method is applied to a pulse with varying envelope, extending the previous work in the field and shows promising numerical results.

References

[1] Vasily Yu. Kharin, Daniel Seipt, and Sergey G. Rykovanov. "Higher-Dimensional Caustics in Nonlinear Compton Scattering". In: *Phys. Rev. Lett.* 120 (4 Jan. 2018), p. 044802.